



# Getting Ahead – Pre-course Work

#ThinkBrock

## A Level Maths and Further Maths

Welcome to **A Level Maths**! In preparation for the start of the course, please complete the attached booklet and have it with you to hand in during your first week of college in September.

These tasks should take you approximately four and a half hours to complete and are designed to give you an introduction to the subject, and the expectations we have for you to complete at least four and a half hours of independent study, per subject, per week.

We recommend that you do one section of the booklet each week, so that you keep your maths knowledge current through the summer, ready for September.

If you are taking **A Level Further Maths** too, we expect you to answer all extension questions; this is optional but recommended for other students.

### Getting Equipped

You will need the following:

- Lined or square A4 paper
- Pen
- Calculator



Brockenhurst  
College

## Bridging work for A Level Mathematics

Name: \_\_\_\_\_

In order to achieve in A Level Mathematics it is **vital** that you have a secure knowledge of GCSE Mathematics content. In particular, you must be **fluent** in the following topics:

- Factorising expressions.
- Solving quadratic equations by factorisation and by the quadratic formula.
- Sketching quadratic graphs.
- Solving linear simultaneous equations using both elimination and substitution.
- Linear inequalities.
- Straight line graphs.
- Parallel and perpendicular lines.
- Pythagoras' theorem.
- Proportion.

We expect that most students will already be confident in the vast majority of these topics.

It is essential that all students spend a significant amount of time practicing these topics at regular intervals between the end of Year 11 and the start of Year 12.

Mathematical fluency does not simply mean that you have met this topic before and think that you remember how to do it. To reach fluency, you must be able to **quickly** and **accurately** recall concepts and methods.

**Mark** your work using the 'Answers' attached at the end of the booklet, checking that you have understood.

If you find that you have made mistakes, **identify** and **correct** these. If you cannot do this, reread the 'Examples' for that specific topic, to ensure that you have not misunderstood a concept.

Students taking Further Maths A Level also have to complete and mark the 'Extend' questions; this is optional but recommended for other students.

Please bring all of your **completed and marked** bridging work to your first maths lesson where it will be checked by your maths lecturer. We expect you to complete the questions on lined or squared paper, showing a **full method** and **working out**.

There will be a baseline assessment covering these topics in the first weeks of Year 12. It is expected that all A Level Mathematics students will demonstrate an excellent understanding of all topics in this assessment.

# Factorising expressions

## A LEVEL LINKS

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form  $ax^2 + bx + c$ , where  $a \neq 0$ .
- To factorise a quadratic equation find two numbers whose sum is  $b$  and whose product is  $ac$ .
- An expression in the form  $x^2 - y^2$  is called the difference of two squares. It factorises to  $(x - y)(x + y)$ .

## Examples

**Example 1** Factorise  $15x^2y^3 + 9x^4y$

$$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$$

The highest common factor is  $3x^2y$ .  
So take  $3x^2y$  outside the brackets and then divide each term by  $3x^2y$  to find the terms in the brackets

**Example 2** Factorise  $4x^2 - 25y^2$

$$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$$

This is the difference of two squares as the two terms can be written as  $(2x)^2$  and  $(5y)^2$

**Example 3** Factorise  $x^2 + 3x - 10$

$$b = 3, ac = -10$$

$$\begin{aligned}\text{So } x^2 + 3x - 10 &= x^2 + 5x - 2x - 10 \\ &= x(x + 5) - 2(x + 5) \\ &= (x + 5)(x - 2)\end{aligned}$$

- 1** Work out the two factors of  $ac = -10$  which add to give  $b = 3$  (5 and  $-2$ )
- 2** Rewrite the  $b$  term ( $3x$ ) using these two factors
- 3** Factorise the first two terms and the last two terms
- 4**  $(x + 5)$  is a factor of both terms

**Example 4** Factorise  $6x^2 - 11x - 10$ 

$b = -11, ac = -60$  So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> <li><b>1</b> Work out the two factors of <math>ac = -60</math> which add to give <math>b = -11</math> (-15 and 4)</li> <li><b>2</b> Rewrite the <math>b</math> term (<math>-11x</math>) using these two factors</li> <li><b>3</b> Factorise the first two terms and the last two terms</li> <li><b>4</b> <math>(2x - 5)</math> is a factor of both terms</li> </ol>
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**Example 5** Simplify  $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ 

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$  For the numerator: $b = -4, ac = -21$  So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$  For the denominator: $b = 9, ac = 18$  So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$  So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	<ol style="list-style-type: none"> <li><b>1</b> Factorise the numerator and the denominator</li> <li><b>2</b> Work out the two factors of <math>ac = -21</math> which add to give <math>b = -4</math> (-7 and 3)</li> <li><b>3</b> Rewrite the <math>b</math> term (<math>-4x</math>) using these two factors</li> <li><b>4</b> Factorise the first two terms and the last two terms</li> <li><b>5</b> <math>(x - 7)</math> is a factor of both terms</li> <li><b>6</b> Work out the two factors of <math>ac = 18</math> which add to give <math>b = 9</math> (6 and 3)</li> <li><b>7</b> Rewrite the <math>b</math> term (<math>9x</math>) using these two factors</li> <li><b>8</b> Factorise the first two terms and the last two terms</li> <li><b>9</b> <math>(x + 3)</math> is a factor of both terms</li> <li><b>10</b> <math>(x + 3)</math> is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1</li> </ol>
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## Practice 1

1 Factorise.

a  $6x^4y^3 - 10x^3y^4$

c  $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

b  $21a^3b^5 + 35a^5b^2$

2 Factorise

a  $x^2 + 7x + 12$

c  $x^2 - 11x + 30$

e  $x^2 - 7x - 18$

g  $x^2 - 3x - 40$

b  $x^2 + 5x - 14$

d  $x^2 - 5x - 24$

f  $x^2 + x - 20$

h  $x^2 + 3x - 28$

3 Factorise

a  $36x^2 - 49y^2$

c  $18a^2 - 200b^2c^2$

b  $4x^2 - 81y^2$

4 Factorise

a  $2x^2 + x - 3$

c  $2x^2 + 7x + 3$

e  $10x^2 + 21x + 9$

b  $6x^2 + 17x + 5$

d  $9x^2 - 15x + 4$

f  $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a  $\frac{2x^2 + 4x}{x^2 - x}$

c  $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e  $\frac{x^2 - x - 12}{x^2 - 4x}$

b  $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d  $\frac{x^2 - 5x}{x^2 - 25}$

f  $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a  $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c  $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b  $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d  $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

## Extend

7 Simplify  $\sqrt{x^2 + 10x + 25}$

8 Simplify  $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

# Solving quadratic equations by factorisation

## A LEVEL LINKS

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- A quadratic equation is an equation in the form  $ax^2 + bx + c = 0$  where  $a \neq 0$ .
- To factorise a quadratic equation find two numbers whose sum is  $b$  and whose products is  $ac$ .
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

## Examples

**Example 1** Solve  $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$  Therefore $x = 0$ or $x = 3$	<ol style="list-style-type: none"><li>1 Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by <math>x</math> as this would lose the solution <math>x = 0</math>.</li><li>2 Factorise the quadratic equation. <math>5x</math> is a common factor.</li><li>3 When two values multiply to make zero, at least one of the values must be zero.</li><li>4 Solve these two equations.</li></ol>
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**Example 2** Solve  $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7, ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$  Therefore $x = -4$ or $x = -3$	<ol style="list-style-type: none"><li>1 Factorise the quadratic equation. Work out the two factors of <math>ac = 12</math> which add to give you <math>b = 7</math>. (4 and 3)</li><li>2 Rewrite the <math>b</math> term (<math>7x</math>) using these two factors.</li><li>3 Factorise the first two terms and the last two terms.</li><li>4 <math>(x + 4)</math> is a factor of both terms.</li><li>5 When two values multiply to make zero, at least one of the values must be zero.</li><li>6 Solve these two equations.</li></ol>
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**Example 3** Solve  $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$  So $(3x + 4) = 0$ or $(3x - 4) = 0$  $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	<ol style="list-style-type: none"><li>Factorise the quadratic equation. This is the difference of two squares as the two terms are <math>(3x)^2</math> and <math>(4)^2</math>.</li><li>When two values multiply to make zero, at least one of the values must be zero.</li><li>Solve these two equations.</li></ol>
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**Example 4** Solve  $2x^2 - 5x - 12 = 0$

$b = -5, ac = -24$  So $2x^2 - 8x + 3x - 12 = 0$  $2x(x - 4) + 3(x - 4) = 0$  $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$  $x = 4$ or $x = -\frac{3}{2}$	<ol style="list-style-type: none"><li>Factorise the quadratic equation. Work out the two factors of <math>ac = -24</math> which add to give you <math>b = -5</math>. (-8 and 3)</li><li>Rewrite the <math>b</math> term (<math>-5x</math>) using these two factors.</li><li>Factorise the first two terms and the last two terms.</li><li><math>(x - 4)</math> is a factor of both terms.</li><li>When two values multiply to make zero, at least one of the values must be zero.</li><li>Solve these two equations.</li></ol>
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## Practice 2a

**1** Solve

**a**  $6x^2 + 4x = 0$

**c**  $x^2 + 7x + 10 = 0$

**e**  $x^2 - 3x - 4 = 0$

**g**  $x^2 - 10x + 24 = 0$

**i**  $x^2 + 3x - 28 = 0$

**k**  $2x^2 - 7x - 4 = 0$

**b**  $28x^2 - 21x = 0$

**d**  $x^2 - 5x + 6 = 0$

**f**  $x^2 + 3x - 10 = 0$

**h**  $x^2 - 36 = 0$

**j**  $x^2 - 6x + 9 = 0$

**l**  $3x^2 - 13x - 10 = 0$

**2** Solve

**a**  $x^2 - 3x = 10$

**c**  $x^2 + 5x = 24$

**e**  $x(x + 2) = 2x + 25$

**g**  $x(3x + 1) = x^2 + 15$

**b**  $x^2 - 3 = 2x$

**d**  $x^2 - 42 = x$

**f**  $x^2 - 30 = 3x - 2$

**h**  $3x(x - 1) = 2(x + 1)$

# Solving quadratic equations by using the formula

## A LEVEL LINKS

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Any quadratic equation of the form  $ax^2 + bx + c = 0$  can be solved using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If  $b^2 - 4ac$  is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for  $a$ ,  $b$  and  $c$ .

## Examples

**Example 5** Solve  $x^2 + 6x + 4 = 0$ . Give your solutions in surd form.

$a = 1, b = 6, c = 4$	<b>1</b> Identify $a$ , $b$ and $c$ and write down the formula.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$ , not just part of it.
$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	<b>2</b> Substitute $a = 1, b = 6, c = 4$ into the formula.
$x = \frac{-6 \pm \sqrt{20}}{2}$	<b>3</b> Simplify. The denominator is 2, but this is only because $a = 1$ . The denominator will not always be 2.
$x = \frac{-6 \pm 2\sqrt{5}}{2}$	<b>4</b> Simplify $\sqrt{20}$ . $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	<b>5</b> Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	<b>6</b> Write down both the solutions.

**Example 6** Solve  $3x^2 - 7x - 2 = 0$ . Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$ $x = \frac{7 \pm \sqrt{73}}{6}$ <p>So <math>x = \frac{7 - \sqrt{73}}{6}</math> or <math>x = \frac{7 + \sqrt{73}}{6}</math></p>	<ol style="list-style-type: none"><li>1 Identify <math>a, b</math> and <math>c</math>, making sure you get the signs right and write down the formula. Remember that <math>-b \pm \sqrt{b^2 - 4ac}</math> is all over <math>2a</math>, not just part of it.</li><li>2 Substitute <math>a = 3, b = -7, c = -2</math> into the formula.</li><li>3 Simplify. The denominator is 6 when <math>a = 3</math>. A common mistake is to always write a denominator of 2.</li><li>4 Write down both the solutions.</li></ol>
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## Practice 2b

3 Solve, giving your solutions in surd form.

**a**  $3x^2 + 6x + 2 = 0$

**b**  $2x^2 - 4x - 7 = 0$

4 Solve the equation  $x^2 - 7x + 2 = 0$

Give your solutions in the form  $\frac{a \pm \sqrt{b}}{c}$ , where  $a, b$  and  $c$  are integers.

5 Solve  $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

## Extend

6 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

**a**  $4x(x - 1) = 3x - 2$

**b**  $10 = (x + 1)^2$

**c**  $x(3x - 1) = 10$

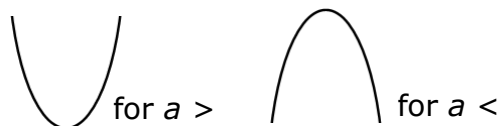
# Sketching quadratic graphs

## A LEVEL LINKS

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- The graph of the quadratic function  $y = ax^2 + bx + c$ , where  $a \neq 0$ , is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the  $y$ -axis substitute  $x = 0$  into the function.
- To find where the curve intersects the  $x$ -axis substitute  $y = 0$  into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.



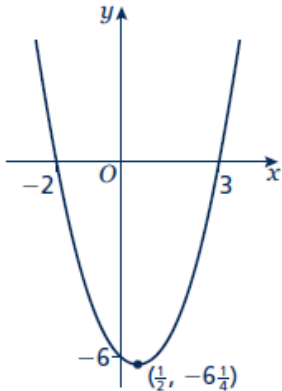
## Examples

**Example 1** Sketch the graph of  $y = x^2$ .

	<p>The graph of <math>y = x^2</math> is a parabola.</p> <p>When <math>x = 0</math>, <math>y = 0</math>.</p> <p><math>a = 1</math> which is greater than zero, so the graph has the shape:</p>
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**Example 2** Sketch the graph of  $y = x^2 - x - 6$ .

<p>When <math>x = 0</math>, <math>y = 0^2 - 0 - 6 = -6</math>          So the graph intersects the <math>y</math>-axis at <math>(0, -6)</math>          When <math>y = 0</math>, <math>x^2 - x - 6 = 0</math>  <math>(x + 2)(x - 3) = 0</math>  <math>x = -2</math> or <math>x = 3</math>          So,          the graph intersects the <math>x</math>-axis at <math>(-2, 0)</math>          and <math>(3, 0)</math></p>	<ol style="list-style-type: none"> <li>Find where the graph intersects the <math>y</math>-axis by substituting <math>x = 0</math>.</li> <li>Find where the graph intersects the <math>x</math>-axis by substituting <math>y = 0</math>.</li> <li>Solve the equation by factorising.</li> <li>Solve <math>(x + 2) = 0</math> and <math>(x - 3) = 0</math>.</li> <li><math>a = 1</math> which is greater than zero, so the graph has the shape:</li> </ol> <p style="text-align: right;"><i>(continued on next page)</i></p>
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$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$ <p>When <math>\left(x - \frac{1}{2}\right)^2 = 0</math>, <math>x = \frac{1}{2}</math> and</p> $y = -\frac{25}{4}$ , so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$ 	<p><b>6</b> To find the turning point, complete the square.</p> <p><b>7</b> The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.</p>
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### Practice 3

- Sketch the graph of  $y = -x^2$ .
- Sketch each graph, labelling where the curve crosses the axes.
 

<b>a</b> $y = (x + 2)(x - 1)$	<b>b</b> $y = x(x - 3)$	<b>c</b> $y = (x + 1)(x + 5)$
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- Sketch each graph, labelling where the curve crosses the axes.
 

<b>a</b> $y = x^2 - x - 6$	<b>b</b> $y = x^2 - 5x + 4$	<b>c</b> $y = x^2 - 4$
<b>d</b> $y = x^2 + 4x$	<b>e</b> $y = 9 - x^2$	<b>f</b> $y = x^2 + 2x - 3$
- Sketch the graph of  $y = 2x^2 + 5x - 3$ , labelling where the curve crosses the axes.

### Extend

- Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.
 

<b>a</b> $y = x^2 - 5x + 6$	<b>b</b> $y = -x^2 + 7x - 12$	<b>c</b> $y = -x^2 + 4x$
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- Sketch the graph of  $y = x^2 + 2x + 1$ . Label where the curve crosses the axes and write down the equation of the line of symmetry.

# Solving linear simultaneous equations using the elimination method

## A LEVEL LINKS

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

## Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

## Examples

**Example 1** Solve the simultaneous equations  $3x + y = 5$  and  $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \\ \text{So } x = 2 \end{array}$ <p>Using <math>x + y = 1</math> <math>2 + y = 1</math> So <math>y = -1</math></p> <p>Check: equation 1: <math>3 \times 2 + (-1) = 5</math> YES equation 2: <math>2 + (-1) = 1</math> YES</p>	<ol style="list-style-type: none"><li>1 Subtract the second equation from the first equation to eliminate the <math>y</math> term.</li><li>2 To find the value of <math>y</math>, substitute <math>x = 2</math> into one of the original equations.</li><li>3 Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</li></ol>
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**Example 2** Solve  $x + 2y = 13$  and  $5x - 2y = 5$  simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x \quad = 18 \\ \text{So } x = 3 \end{array}$ <p>Using <math>x + 2y = 13</math> <math>3 + 2y = 13</math> So <math>y = 5</math></p> <p>Check: equation 1: <math>3 + 2 \times 5 = 13</math> YES equation 2: <math>5 \times 3 - 2 \times 5 = 5</math> YES</p>	<ol style="list-style-type: none"><li>1 Add the two equations together to eliminate the <math>y</math> term.</li><li>2 To find the value of <math>y</math>, substitute <math>x = 3</math> into one of the original equations.</li><li>3 Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</li></ol>
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**Example 3** Solve  $2x + 3y = 2$  and  $5x + 4y = 12$  simultaneously.

$\begin{array}{r} (2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8 \\ (5x + 4y = 12) \times 3 \rightarrow \underline{15x + 12y = 36} \\ \phantom{(5x + 4y = 12) \times 3 \rightarrow} 7x = 28 \end{array}$	<b>1</b> Multiply the first equation by 4 and the second equation by 3 to make the coefficient of $y$ the same for both equations. Then subtract the first equation from the second equation to eliminate the $y$ term.
So $x = 4$	
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$	<b>2</b> To find the value of $y$ , substitute $x = 4$ into one of the original equations.
So $y = -2$	
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	<b>3</b> Substitute the values of $x$ and $y$ into both equations to check your answers.

## Practice 4a

Solve these simultaneous equations.

**1**  $4x + y = 8$   
 $x + y = 5$

**2**  $3x + y = 7$   
 $3x + 2y = 5$

**3**  $4x + y = 3$   
 $3x - y = 11$

**4**  $3x + 4y = 7$   
 $x - 4y = 5$

**5**  $2x + y = 11$   
 $x - 3y = 9$

**6**  $2x + 3y = 11$   
 $3x + 2y = 4$

# Solving linear simultaneous equations using the substitution method

## A LEVEL LINKS

**Scheme of work:** 1c. Equations – quadratic/linear simultaneous

**Textbook:** Pure Year 1, 3.1 Linear simultaneous equations

## Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

## Examples

**Example 4** Solve the simultaneous equations  $y = 2x + 1$  and  $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	<b>1</b> Substitute $2x + 1$ for $y$ into the second equation.
$5x + 6x + 3 = 14$	<b>2</b> Expand the brackets and simplify.
$11x + 3 = 14$	<b>3</b> Work out the value of $x$ .
$11x = 11$	
So $x = 1$	
Using $y = 2x + 1$	<b>4</b> To find the value of $y$ , substitute $x = 1$ into one of the original equations.
$y = 2 \times 1 + 1$	
So $y = 3$	
Check:	<b>5</b> Substitute the values of $x$ and $y$ into both equations to check your answers.
equation 1: $3 = 2 \times 1 + 1$ YES	
equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	

**Example 5** Solve  $2x - y = 16$  and  $4x + 3y = -3$  simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ $\text{So } x = 4\frac{1}{2}$ <p>Using <math>y = 2x - 16</math></p> $y = 2 \times 4\frac{1}{2} - 16$ $\text{So } y = -7$ <p>Check:</p> <p>equation 1: <math>2 \times 4\frac{1}{2} - (-7) = 16</math>    YES</p> <p>equation 2: <math>4 \times 4\frac{1}{2} + 3 \times (-7) = -3</math> YES</p>	<ol style="list-style-type: none"> <li><b>1</b> Rearrange the first equation.</li> <li><b>2</b> Substitute <math>2x - 16</math> for <math>y</math> into the second equation.</li> <li><b>3</b> Expand the brackets and simplify.</li> <li><b>4</b> Work out the value of <math>x</math>.</li> <li><b>5</b> To find the value of <math>y</math>, substitute <math>x = 4\frac{1}{2}</math> into one of the original equations.</li> <li><b>6</b> Substitute the values of <math>x</math> and <math>y</math> into both equations to check your answers.</li> </ol>
--	---

## Practice 4b

Solve these simultaneous equations.

**7**  $y = x - 4$   
 $2x + 5y = 43$

**8**  $y = 2x - 3$   
 $5x - 3y = 11$

**9**  $2y = 4x + 5$   
 $9x + 5y = 22$

**10**  $2x = y - 2$   
 $8x - 5y = -11$

**11**  $3x + 4y = 8$   
 $2x - y = -13$

**12**  $3y = 4x - 7$   
 $2y = 3x - 4$

**13**  $3x = y - 1$   
 $2y - 2x = 3$

**14**  $3x + 2y + 1 = 0$   
 $4y = 8 - x$

## Extend

**15** Solve the simultaneous equations  $3x + 5y - 20 = 0$  and  $2(x + y) = \frac{3(y - x)}{4}$ .

# Linear inequalities

## A LEVEL LINKS

**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

## Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g.  $<$  becomes  $>$ .

## Examples

**Example 1** Solve  $-8 \leq 4x < 16$

$\begin{aligned} -8 &\leq 4x < 16 \\ -2 &\leq x < 4 \end{aligned}$	Divide all three terms by 4.
--	------------------------------

**Example 2** Solve  $4 \leq 5x < 10$

$\begin{aligned} 4 &\leq 5x < 10 \\ \frac{4}{5} &\leq x < 2 \end{aligned}$	Divide all three terms by 5.
--	------------------------------

**Example 3** Solve  $2x - 5 < 7$

$\begin{aligned} 2x - 5 &< 7 \\ 2x &< 12 \\ x &< 6 \end{aligned}$	<ol style="list-style-type: none"><li>1 Add 5 to both sides.</li><li>2 Divide both sides by 2.</li></ol>
---	--

**Example 4** Solve  $2 - 5x \geq -8$

$\begin{aligned} 2 - 5x &\geq -8 \\ -5x &\geq -10 \\ x &\leq 2 \end{aligned}$	<ol style="list-style-type: none"><li>1 Subtract 2 from both sides.</li><li>2 Divide both sides by <math>-5</math>. Remember to reverse the inequality when dividing by a negative number.</li></ol>
---	--

**Example 5** Solve  $4(x - 2) > 3(9 - x)$

$\begin{aligned} 4(x - 2) &> 3(9 - x) \\ 4x - 8 &> 27 - 3x \\ 7x - 8 &> 27 \\ 7x &> 35 \\ x &> 5 \end{aligned}$	<ol style="list-style-type: none"><li>1 Expand the brackets.</li><li>2 Add <math>3x</math> to both sides.</li><li>3 Add 8 to both sides.</li><li>4 Divide both sides by 7.</li></ol>
---	--

## Practice 5

1 Solve these inequalities.

**a**  $4x > 16$

**b**  $5x - 7 \leq 3$

**c**  $1 \geq 3x + 4$

**d**  $5 - 2x < 12$

**e**  $\frac{x}{2} \geq 5$

**f**  $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

**a**  $\frac{x}{5} < -4$

**b**  $10 \geq 2x + 3$

**c**  $7 - 3x > -5$

3 Solve

**a**  $2 - 4x \geq 18$

**b**  $3 \leq 7x + 10 < 45$

**c**  $6 - 2x \geq 4$

**d**  $4x + 17 < 2 - x$

**e**  $4 - 5x < -3x$

**f**  $-4x \geq 24$

4 Solve these inequalities.

**a**  $3t + 1 < t + 6$

**b**  $2(3n - 1) \geq n + 5$

5 Solve.

**a**  $3(2 - x) > 2(4 - x) + 4$

**b**  $5(4 - x) > 3(5 - x) + 2$

## Extend

6 Find the set of values of  $x$  for which  $2x + 1 > 11$  and  $4x - 2 > 16 - 2x$ .

# Straight line graphs

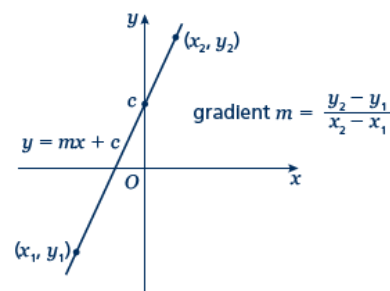
## A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

## Key points

- A straight line has the equation  $y = mx + c$ , where  $m$  is the gradient and  $c$  is the  $y$ -intercept (where  $x = 0$ ).
- The equation of a straight line can be written in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.
- When given the coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$  of two points on a line the gradient is calculated using the

$$\text{formula } m = \frac{y_2 - y_1}{x_2 - x_1}$$



## Examples

**Example 1** A straight line has gradient  $-\frac{1}{2}$  and  $y$ -intercept 3.

Write the equation of the line in the form  $ax + by + c = 0$ .

$$m = -\frac{1}{2} \text{ and } c = 3$$

$$\text{So } y = -\frac{1}{2}x + 3$$

$$\frac{1}{2}x + y - 3 = 0$$

$$x + 2y - 6 = 0$$

- 1 A straight line has equation  $y = mx + c$ . Substitute the gradient and  $y$ -intercept given in the question into this equation.
- 2 Rearrange the equation so all the terms are on one side and 0 is on the other side.
- 3 Multiply both sides by 2 to eliminate the denominator.

**Example 2** Find the gradient and the  $y$ -intercept of the line with the equation  $3y - 2x + 4 = 0$ .

$$3y - 2x + 4 = 0$$

$$3y = 2x - 4$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

$$\text{Gradient} = m = \frac{2}{3}$$

$$\text{y-intercept} = c = -\frac{4}{3}$$

- 1 Make  $y$  the subject of the equation.
- 2 Divide all the terms by three to get the equation in the form  $y = \dots$
- 3 In the form  $y = mx + c$ , the gradient is  $m$  and the  $y$ -intercept is  $c$ .

**Example 3** Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	<ol style="list-style-type: none"> <li><b>1</b> Substitute the gradient given in the question into the equation of a straight line <math>y = mx + c</math>.</li> <li><b>2</b> Substitute the coordinates <math>x = 5</math> and <math>y = 13</math> into the equation.</li> <li><b>3</b> Simplify and solve the equation.</li> <li><b>4</b> Substitute <math>c = -2</math> into the equation <math>y = 3x + c</math></li> </ol>
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**Example 4** Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2, x_2 = 8, y_1 = 4 \text{ and } y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$ $y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	<ol style="list-style-type: none"> <li><b>1</b> Substitute the coordinates into the equation <math>m = \frac{y_2 - y_1}{x_2 - x_1}</math> to work out the gradient of the line.</li> <li><b>2</b> Substitute the gradient into the equation of a straight line <math>y = mx + c</math>.</li> <li><b>3</b> Substitute the coordinates of either point into the equation.</li> <li><b>4</b> Simplify and solve the equation.</li> <li><b>5</b> Substitute <math>c = 3</math> into the equation <math>y = \frac{1}{2}x + c</math></li> </ol>
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## Practice 6

**1** Find the gradient and the y-intercept of the following equations.

- |                            |                                  |
|----------------------------|----------------------------------|
| <b>a</b> $y = 3x + 5$      | <b>b</b> $y = -\frac{1}{2}x - 7$ |
| <b>c</b> $2y = 4x - 3$     | <b>d</b> $x + y = 5$             |
| <b>e</b> $2x - 3y - 7 = 0$ | <b>f</b> $5x + y - 4 = 0$        |

**2** Copy and complete the table, giving the equation of the line in the form  $y = mx + c$ .

Gradient	y-intercept	Equation of the line
5	0	
-3	2	
4	-7	

- 3** Find, in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers, an equation for each of the lines with the following gradients and  $y$ -intercepts.
- a** gradient  $-\frac{1}{2}$ ,  $y$ -intercept  $-7$       **b** gradient  $2$ ,  $y$ -intercept  $0$
- c** gradient  $\frac{2}{3}$ ,  $y$ -intercept  $4$       **d** gradient  $-1.2$ ,  $y$ -intercept  $-2$
- 4** Write an equation for the line which passes through the point  $(2, 5)$  and has gradient  $4$ .
- 5** Write an equation for the line which passes through the point  $(6, 3)$  and has gradient  $-\frac{2}{3}$ .
- 6** Write an equation for the line passing through each of the following pairs of points.
- a**  $(4, 5)$ ,  $(10, 17)$       **b**  $(0, 6)$ ,  $(-4, 8)$
- c**  $(-1, -7)$ ,  $(5, 23)$       **d**  $(3, 10)$ ,  $(4, 7)$

## Extend

- 7** The equation of a line is  $2y + 3x - 6 = 0$ .  
Write as much information as possible about this line.

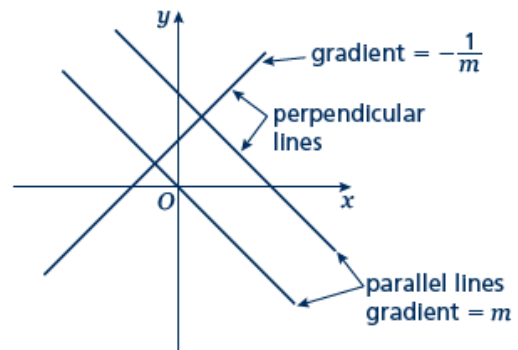
# Parallel and perpendicular lines

## A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

## Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation  $y = mx + c$  has gradient  $-\frac{1}{m}$ .



## Examples

**Example 1** Find the equation of the line parallel to  $y = 2x + 4$  which passes through the point  $(4, 9)$ .

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	<ol style="list-style-type: none"> <li>1 As the lines are parallel they have the same gradient.</li> <li>2 Substitute <math>m = 2</math> into the equation of a straight line <math>y = mx + c</math>.</li> <li>3 Substitute the coordinates into the equation <math>y = 2x + c</math></li> <li>4 Simplify and solve the equation.</li> <li>5 Substitute <math>c = 1</math> into the equation <math>y = 2x + c</math></li> </ol>
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**Example 2** Find the equation of the line perpendicular to  $y = 2x - 3$  which passes through the point  $(-2, 5)$ .

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	<ol style="list-style-type: none"> <li>1 As the lines are perpendicular, the gradient of the perpendicular line is <math>-\frac{1}{m}</math>.</li> <li>2 Substitute <math>m = -\frac{1}{2}</math> into <math>y = mx + c</math>.</li> <li>3 Substitute the coordinates <math>(-2, 5)</math> into the equation <math>y = -\frac{1}{2}x + c</math></li> <li>4 Simplify and solve the equation.</li> <li>5 Substitute <math>c = 4</math> into <math>y = -\frac{1}{2}x + c</math>.</li> </ol>
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**Example 3** A line passes through the points  $(0, 5)$  and  $(9, -1)$ . Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0, x_2 = 9, y_1 = 5 \text{ and } y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0}$ $= \frac{-6}{9} = -\frac{2}{3}$ $-\frac{1}{m} = \frac{3}{2}$ $y = \frac{3}{2}x + c$ $\text{Midpoint} = \left( \frac{0+9}{2}, \frac{5+(-1)}{2} \right) = \left( \frac{9}{2}, 2 \right)$ $2 = \frac{3}{2} \times \frac{9}{2} + c$ $c = -\frac{19}{4}$ $y = \frac{3}{2}x - \frac{19}{4}$	<p><b>1</b> Substitute the coordinates into the equation <math>m = \frac{y_2 - y_1}{x_2 - x_1}</math> to work out the gradient of the line.</p> <p><b>2</b> As the lines are perpendicular, the gradient of the perpendicular line is <math>-\frac{1}{m}</math>.</p> <p><b>3</b> Substitute the gradient into the equation <math>y = mx + c</math>.</p> <p><b>4</b> Work out the coordinates of the midpoint of the line.</p> <p><b>5</b> Substitute the coordinates of the midpoint into the equation.</p> <p><b>6</b> Simplify and solve the equation.</p> <p><b>7</b> Substitute <math>c = -\frac{19}{4}</math> into the equation</p> $y = \frac{3}{2}x + c.$
--	--

## Practice 7

- Find the equation of the line parallel to each of the given lines and which passes through each of the given points.
 

<b>a</b> $y = 3x + 1$ (3, 2)	<b>b</b> $y = 3 - 2x$ (1, 3)
<b>c</b> $2x + 4y + 3 = 0$ (6, -3)	<b>d</b> $2y - 3x + 2 = 0$ (8, 20)
- Find the equation of the line perpendicular to  $y = \frac{1}{2}x - 3$  which passes through the point  $(-5, 3)$ .
- Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.
 

<b>a</b> $y = 2x - 6$ (4, 0)	<b>b</b> $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)
<b>c</b> $x - 4y - 4 = 0$ (5, 15)	<b>d</b> $5y + 2x - 5 = 0$ (6, 7)

- 4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.
- a  $(4, 3), (-2, -9)$                       b  $(0, 3), (-10, 8)$

## Extend

- 5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a  $y = 2x + 3$   
 $y = 2x - 7$

b  $y = 3x$   
 $2x + y - 3 = 0$

c  $y = 4x - 3$   
 $4y + x = 2$

d  $3x - y + 5 = 0$   
 $x + 3y = 1$

e  $2x + 5y - 1 = 0$   
 $y = 2x + 7$

f  $2x - y = 6$   
 $6x - 3y + 3 = 0$

- 6 The straight line  $L_1$  passes through the points  $A$  and  $B$  with coordinates  $(-4, 4)$  and  $(2, 1)$ , respectively.

a Find the equation of  $L_1$  in the form  $ax + by + c = 0$

The line  $L_2$  is parallel to the line  $L_1$  and passes through the point  $C$  with coordinates  $(-8, 3)$ .

b Find the equation of  $L_2$  in the form  $ax + by + c = 0$

The line  $L_3$  is perpendicular to the line  $L_1$  and passes through the origin.

c Find an equation of  $L_3$

# Pythagoras' theorem

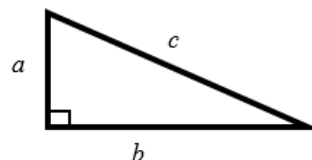
## A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

## Key points

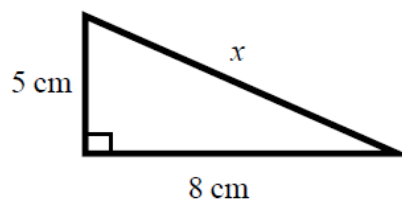
- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

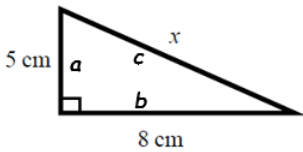
$$c^2 = a^2 + b^2$$



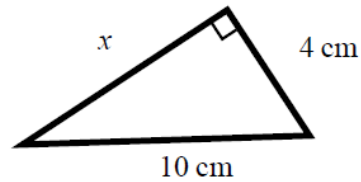
## Examples

**Example 1** Calculate the length of the hypotenuse.  
Give your answer to 3 significant figures.



<p><math>c^2 = a^2 + b^2</math></p>  <p><math>x^2 = 5^2 + 8^2</math> <math>x^2 = 25 + 64</math> <math>x^2 = 89</math> <math>x = \sqrt{89}</math></p> <p><math>x = 9.433\ 981\ 13\dots</math> <math>x = 9.43\text{ cm}</math></p>	<ol style="list-style-type: none"><li>1 Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse <math>c</math> and the other two sides <math>a</math> and <math>b</math>.</li><li>2 Substitute the values of <math>a</math>, <math>b</math> and <math>c</math> into the formula for Pythagoras' theorem.</li><li>3 Use a calculator to find the square root.</li><li>4 Round your answer to 3 significant figures and write the units with your answer.</li></ol>
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**Example 2** Calculate the length  $x$ .  
Give your answer in surd form.

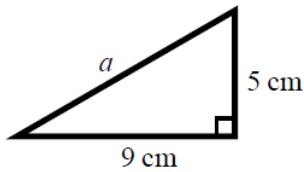


$c^2 = a^2 + b^2$ $10^2 = x^2 + 4^2$ $100 = x^2 + 16$ $x^2 = 84$ $x = \sqrt{84}$ $x = 2\sqrt{21} \text{ cm}$	<ol style="list-style-type: none"> <li><b>1</b> Always start by stating the formula for Pythagoras' theorem.</li> <li><b>2</b> Substitute the values of <math>a</math>, <math>b</math> and <math>c</math> into the formula for Pythagoras' theorem.</li> <li><b>3</b> Simplify the surd where possible and write the units in your answer.</li> </ol>
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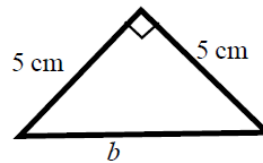
## Practice 8

**1** Work out the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.

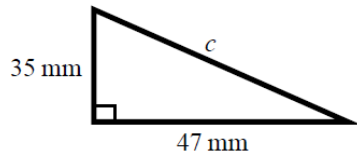
**a**



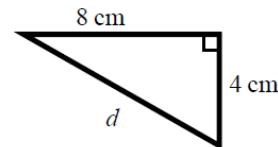
**b**



**c**

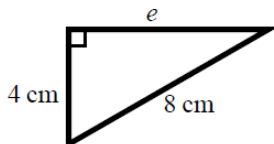


**d**

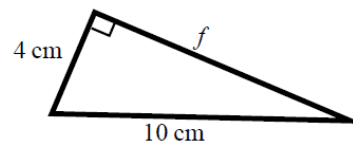


**2** Work out the length of the unknown side in each triangle.  
Give your answers in surd form.

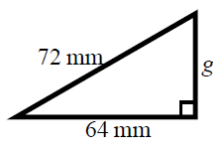
**a**



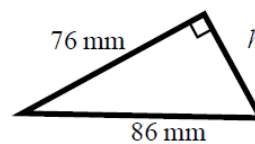
**b**



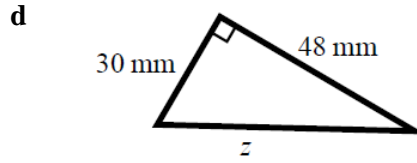
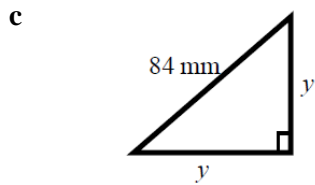
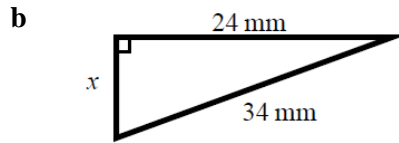
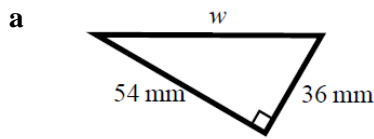
**c**



**d**



- 3 Work out the length of the unknown side in each triangle.  
Give your answers in surd form.

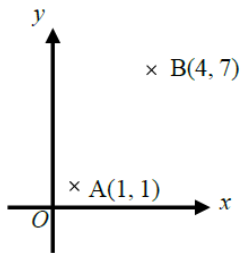


- 4 A rectangle has length 84 mm and width 45 mm.  
Calculate the length of the diagonal of the rectangle.  
Give your answer correct to 3 significant figures.

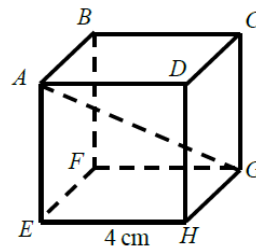
## Extend

- 5 A yacht is 40 km due North of a lighthouse.  
A rescue boat is 50 km due East of the same lighthouse.  
Work out the distance between the yacht and the rescue boat.  
Give your answer correct to 3 significant figures.

- 6 Points A and B are shown on the diagram.  
Work out the length of the line AB.  
Give your answer in surd form.



- 7 A cube has length 4 cm.  
Work out the length of the diagonal  $AG$ .  
Give your answer in surd form.



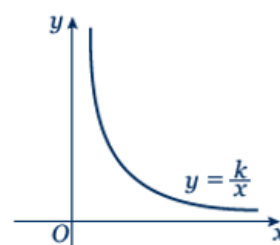
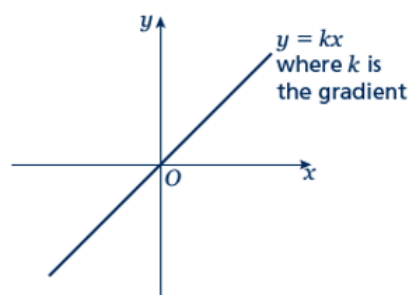
# Proportion

## A LEVEL LINKS

**Scheme of work:** 2a. Straight-line graphs, parallel/perpendicular, length and area problems

## Key points

- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- 'y is directly proportional to x' is written as  $y \propto x$ .  
If  $y \propto x$  then  $y = kx$ , where  $k$  is a constant.
- When  $x$  is directly proportional to  $y$ , the graph is a straight line passing through the origin.
- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- 'y is inversely proportional to x' is written as  $y \propto \frac{1}{x}$ .  
If  $y \propto \frac{1}{x}$  then  $y = \frac{k}{x}$ , where  $k$  is a constant.
- When  $x$  is inversely proportional to  $y$  the graph is the same shape as the graph of  $y = \frac{1}{x}$



## Examples

**Example 1**  $y$  is directly proportional to  $x$ .

When  $y = 16$ ,  $x = 5$ .

- Find  $x$  when  $y = 30$ .
- Sketch the graph of the formula.

**a**  $y \propto x$

$$y = kx$$
$$16 = k \times 5$$

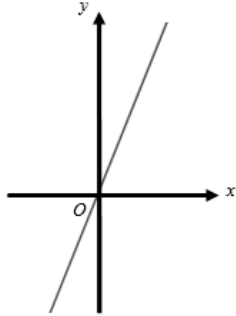
$$k = 3.2$$

$$y = 3.2x$$

When  $y = 30$ ,

$$30 = 3.2 \times x$$
$$x = 9.375$$

- Write  $y$  is directly proportional to  $x$ , using the symbol  $\propto$ .
- Write the equation using  $k$ .
- Substitute  $y = 16$  and  $x = 5$  into  $y = kx$ .
- Solve the equation to find  $k$ .
- Substitute the value of  $k$  back into the equation  $y = kx$ .
- Substitute  $y = 30$  into  $y = 3.2x$  and solve to find  $x$  when  $y = 30$ .

<p><b>b</b></p> 	<p><b>7</b> The graph of <math>y = 3.2x</math> is a straight line passing through <math>(0, 0)</math> with a gradient of 3.2.</p>
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**Example 2**  $y$  is directly proportional to  $x^2$ .  
When  $x = 3$ ,  $y = 45$ .

**a** Find  $y$  when  $x = 5$ .  
**b** Find  $x$  when  $y = 20$ .

<p><b>a</b> <math>y \propto x^2</math></p> $y = kx^2$ $45 = k \times 3^2$ $k = 5$ $y = 5x^2$ <p>When <math>x = 5</math>,</p> $y = 5 \times 5^2$ $y = 125$ <p><b>b</b> <math>20 = 5 \times x^2</math></p> $x^2 = 4$ $x = \pm 2$	<ol style="list-style-type: none"> <li><b>1</b> Write <math>y</math> is directly proportional to <math>x^2</math>, using the symbol <math>\propto</math>.</li> <li><b>2</b> Write the equation using <math>k</math>.</li> <li><b>3</b> Substitute <math>y = 45</math> and <math>x = 3</math> into <math>y = kx^2</math>.</li> <li><b>4</b> Solve the equation to find <math>k</math>.</li> <li><b>5</b> Substitute the value of <math>k</math> back into the equation <math>y = kx^2</math>.</li> <li><b>6</b> Substitute <math>x = 5</math> into <math>y = 5x^2</math> and solve to find <math>y</math> when <math>x = 5</math>.</li> <li><b>7</b> Substitute <math>y = 20</math> into <math>y = 5x^2</math> and solve to find <math>x</math> when <math>y = 4</math>.</li> </ol>
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**Example 3**  $P$  is inversely proportional to  $Q$ .  
When  $P = 100$ ,  $Q = 10$ .  
Find  $Q$  when  $P = 20$ .

$P \propto \frac{1}{Q}$ $P = \frac{k}{Q}$ $100 = \frac{k}{10}$ $k = 1000$ $P = \frac{1000}{Q}$ $20 = \frac{1000}{Q}$	<ol style="list-style-type: none"> <li><b>1</b> Write <math>P</math> is inversely proportional to <math>Q</math>, using the symbol <math>\propto</math>.</li> <li><b>2</b> Write the equation using <math>k</math>.</li> <li><b>3</b> Substitute <math>P = 100</math> and <math>Q = 10</math>.</li> <li><b>4</b> Solve the equation to find <math>k</math>.</li> <li><b>5</b> Substitute the value of <math>k</math> into <math>P = \frac{k}{Q}</math></li> <li><b>6</b> Substitute <math>P = 20</math> into <math>P = \frac{1000}{Q}</math> and solve to find <math>Q</math> when <math>P = 20</math>.</li> </ol>
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$Q = \frac{1000}{20} = 50$	
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## Practice 9

- Paul gets paid an hourly rate. The amount of pay (£ $P$ ) is directly proportional to the number of hours ( $h$ ) he works.  
When he works 8 hours he is paid £56.  
If Paul works for 11 hours, how much is he paid?
- $x$  is directly proportional to  $y$ .  
 $x = 35$  when  $y = 5$ .
  - Find a formula for  $x$  in terms of  $y$ .
  - Sketch the graph of the formula.
  - Find  $x$  when  $y = 13$ .
  - Find  $y$  when  $x = 63$ .
- $Q$  is directly proportional to the square of  $Z$ .  
 $Q = 48$  when  $Z = 4$ .
  - Find a formula for  $Q$  in terms of  $Z$ .
  - Sketch the graph of the formula.
  - Find  $Q$  when  $Z = 5$ .
  - Find  $Z$  when  $Q = 300$ .
- $y$  is directly proportional to the square of  $x$ .  
 $x = 2$  when  $y = 10$ .
  - Find a formula for  $y$  in terms of  $x$ .
  - Sketch the graph of the formula.
  - Find  $x$  when  $y = 90$ .
- $B$  is directly proportional to the square root of  $C$ .  
 $C = 25$  when  $B = 10$ .
  - Find  $B$  when  $C = 64$ .
  - Find  $C$  when  $B = 20$ .
- $C$  is directly proportional to  $D$ .  
 $C = 100$  when  $D = 150$ .  
Find  $C$  when  $D = 450$ .
- $y$  is directly proportional to  $x$ .  
 $x = 27$  when  $y = 9$ .  
Find  $x$  when  $y = 3.7$ .
- $m$  is proportional to the cube of  $n$ .  
 $m = 54$  when  $n = 3$ .  
Find  $n$  when  $m = 250$ .

## Extend

- 9**  $s$  is inversely proportional to  $t$ .
- Given that  $s = 2$  when  $t = 2$ , find a formula for  $s$  in terms of  $t$ .
  - Sketch the graph of the formula.
  - Find  $t$  when  $s = 1$ .
- 10**  $a$  is inversely proportional to  $b$ .  
 $a = 5$  when  $b = 20$ .
- Find  $a$  when  $b = 50$ .
  - Find  $b$  when  $a = 10$ .
- 11**  $v$  is inversely proportional to  $w$ .  
 $w = 4$  when  $v = 20$ .
- Find a formula for  $v$  in terms of  $w$ .
  - Sketch the graph of the formula.
  - Find  $w$  when  $v = 2$ .
- 12**  $L$  is inversely proportional to  $W$ .  
 $L = 12$  when  $W = 3$ .  
Find  $W$  when  $L = 6$ .
- 13**  $s$  is inversely proportional to  $t$ .  
 $s = 6$  when  $t = 12$ .
- Find  $s$  when  $t = 3$ .
  - Find  $t$  when  $s = 18$ .
- 14**  $y$  is inversely proportional to  $x^2$ .  
 $y = 4$  when  $x = 2$ .  
Find  $y$  when  $x = 4$ .
- 15**  $y$  is inversely proportional to the square root of  $x$ .  
 $x = 25$  when  $y = 1$ .  
Find  $x$  when  $y = 5$ .
- 16**  $a$  is inversely proportional to  $b$ .  
 $a = 0.05$  when  $b = 4$ .
- Find  $a$  when  $b = 2$ .
  - Find  $b$  when  $a = 2$ .

## Answers 1

- 1**   **a**    $2x^3y^3(3x - 5y)$                       **b**    $7a^3b^2(3b^3 + 5a^2)$   
      **c**    $5x^2y^2(5 - 2x + 3y)$
- 2**   **a**    $(x + 3)(x + 4)$                       **b**    $(x + 7)(x - 2)$   
      **c**    $(x - 5)(x - 6)$                       **d**    $(x - 8)(x + 3)$   
      **e**    $(x - 9)(x + 2)$                       **f**    $(x + 5)(x - 4)$   
      **g**    $(x - 8)(x + 5)$                       **h**    $(x + 7)(x - 4)$
- 3**   **a**    $(6x - 7y)(6x + 7y)$                       **b**    $(2x - 9y)(2x + 9y)$   
      **c**    $2(3a - 10bc)(3a + 10bc)$
- 4**   **a**    $(x - 1)(2x + 3)$                       **b**    $(3x + 1)(2x + 5)$   
      **c**    $(2x + 1)(x + 3)$                       **d**    $(3x - 1)(3x - 4)$   
      **e**    $(5x + 3)(2x + 3)$                       **f**    $2(3x - 2)(2x - 5)$
- 5**   **a**    $\frac{2(x+2)}{x-1}$                                       **b**    $\frac{x}{x-1}$   
      **c**    $\frac{x+2}{x}$     **d**    $\frac{x}{x+5}$   
      **e**    $\frac{x+3}{x}$     **f**    $\frac{x}{x-5}$
- 6**   **a**    $\frac{3x+4}{x+7}$     **b**    $\frac{2x+3}{3x-2}$   
      **c**    $\frac{2-5x}{2x-3}$     **d**    $\frac{3x+1}{x+4}$
- 7**    $(x + 5)$
- 8**    $\frac{4(x+2)}{x-2}$

## Answers 2

1 a  $x = 0$  or  $x = -\frac{2}{3}$

c  $x = -5$  or  $x = -2$

e  $x = -1$  or  $x = 4$

g  $x = 4$  or  $x = 6$

i  $x = -7$  or  $x = 4$

k  $x = -\frac{1}{2}$  or  $x = 4$

b  $x = 0$  or  $x = \frac{3}{4}$

d  $x = 2$  or  $x = 3$

f  $x = -5$  or  $x = 2$

h  $x = -6$  or  $x = 6$

j  $x = 3$

l  $x = -\frac{2}{3}$  or  $x = 5$

2 a  $x = -2$  or  $x = 5$

c  $x = -8$  or  $x = 3$

e  $x = -5$  or  $x = 5$

g  $x = -3$  or  $x = 2\frac{1}{2}$

b  $x = -1$  or  $x = 3$

d  $x = -6$  or  $x = 7$

f  $x = -4$  or  $x = 7$

h  $x = -\frac{1}{3}$  or  $x = 2$

3 a  $x = -1 + \frac{\sqrt{3}}{3}$  or  $x = -1 - \frac{\sqrt{3}}{3}$

b  $x = 1 + \frac{3\sqrt{2}}{2}$  or  $x = 1 - \frac{3\sqrt{2}}{2}$

4  $x = \frac{7 + \sqrt{41}}{2}$  or  $x = \frac{7 - \sqrt{41}}{2}$

5  $x = \frac{-3 + \sqrt{89}}{20}$  or  $x = \frac{-3 - \sqrt{89}}{20}$

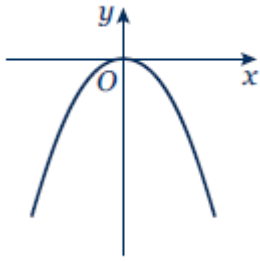
6 a  $x = \frac{7 + \sqrt{17}}{8}$  or  $x = \frac{7 - \sqrt{17}}{8}$

b  $x = -1 + \sqrt{10}$  or  $x = -1 - \sqrt{10}$

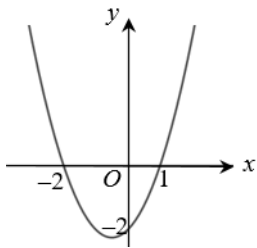
c  $x = -1\frac{2}{3}$  or  $x = 2$

# Answers 3

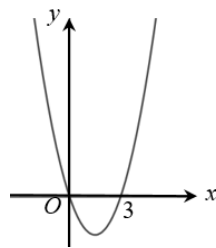
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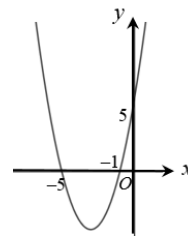
2 a



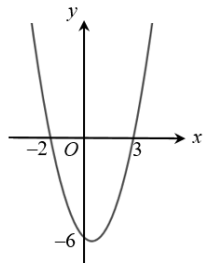
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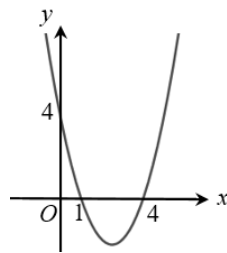
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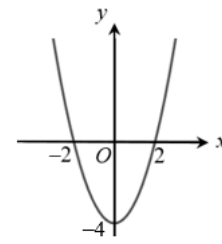
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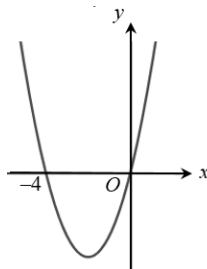
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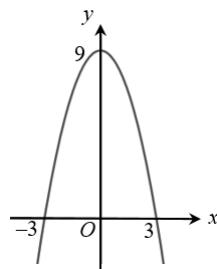
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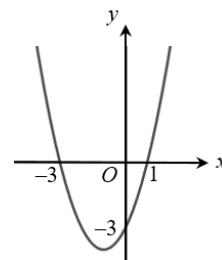
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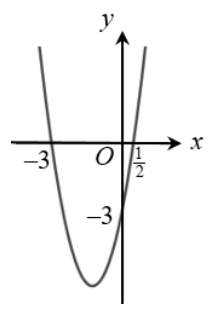
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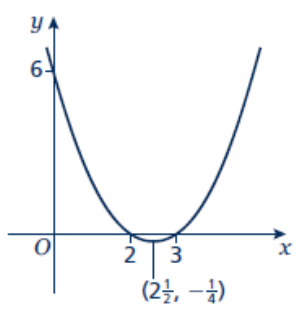
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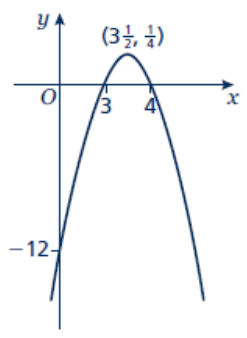
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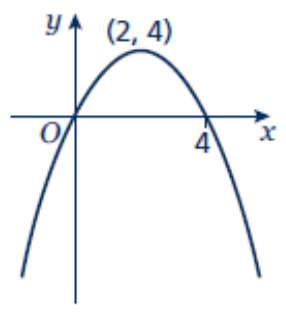
5 a



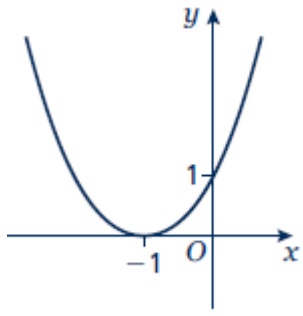
b



c



6



Line of symmetry at  $x = -1$ .

## Answers 4

1  $x = 1, y = 4$

2  $x = 3, y = -2$

3  $x = 2, y = -5$

4  $x = 3, y = -\frac{1}{2}$

5  $x = 6, y = -1$

6  $x = -2, y = 5$

7  $x = 9, y = 5$

8  $x = -2, y = -7$

9  $x = \frac{1}{2}, y = 3\frac{1}{2}$

10  $x = \frac{1}{2}, y = 3$

11  $x = -4, y = 5$

12  $x = -2, y = -5$

13  $x = \frac{1}{4}, y = 1\frac{3}{4}$

14  $x = -2, y = 2\frac{1}{2}$

15  $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

## Answers 5

- 1**   **a**    $x > 4$                       **b**    $x \leq 2$                       **c**    $x \leq -1$   
      **d**    $x > -\frac{7}{2}$                       **e**    $x \geq 10$                       **f**    $x < -15$
- 2**   **a**    $x < -20$                       **b**    $x \leq 3.5$                       **c**    $x < 4$
- 3**   **a**    $x \leq -4$                       **b**    $-1 \leq x < 5$                       **c**    $x \leq 1$   
      **d**    $x < -3$                       **e**    $x > 2$                       **f**    $x \leq -6$
- 4**   **a**    $t < \frac{5}{2}$                       **b**    $n \geq \frac{7}{5}$
- 5**   **a**    $x < -6$                       **b**    $x < \frac{3}{2}$
- 6**    $x > 5$  (which also satisfies  $x > 3$ )

## Answers 6

- 1 a**  $m = 3, c = 5$                       **b**  $m = -\frac{1}{2}, c = -7$   
**c**  $m = 2, c = -\frac{3}{2}$                       **d**  $m = -1, c = 5$   
**e**  $m = \frac{2}{3}, c = -\frac{7}{3}$  or  $-2\frac{1}{3}$                       **f**  $m = -5, c = 4$

**2**

Gradient	y-intercept	Equation of the line
5	0	$y = 5x$
-3	2	$y = -3x + 2$
4	-7	$y = 4x - 7$

- 3 a**  $x + 2y + 14 = 0$                       **b**  $2x - y = 0$   
**c**  $2x - 3y + 12 = 0$                       **d**  $6x + 5y + 10 = 0$

**4**  $y = 4x - 3$

**5**  $y = -\frac{2}{3}x + 7$

**6 a**  $y = 2x - 3$                       **b**  $y = -\frac{1}{2}x + 6$

**c**  $y = 5x - 2$                       **d**  $y = -3x + 19$

**7**  $y = -\frac{3}{2}x + 3$ , the gradient is  $-\frac{3}{2}$  and the y-intercept is 3. The line intercepts the axes at  $(0, 3)$  and  $(2, 0)$ . Students may sketch the line or give coordinates that lie on the line such as  $\left(1, \frac{3}{2}\right)$  or  $(4, -3)$ .

## Answers 7

**1 a**  $y = 3x - 7$

**c**  $y = -\frac{1}{2}x$

**b**  $y = -2x + 5$

**d**  $y = \frac{3}{2}x + 8$

**2**  $y = -2x - 7$

**3 a**  $y = -\frac{1}{2}x + 2$

**c**  $y = -4x + 35$

**b**  $y = 3x + 7$

**d**  $y = \frac{5}{2}x - 8$

**4 a**  $y = -\frac{1}{2}x$

**b**  $y = 2x$

**5 a** Parallel

**d** Perpendicular

**b** Neither

**e** Neither

**c** Perpendicular

**f** Parallel

**6 a**  $x + 2y - 4 = 0$

**b**  $x + 2y + 2 = 0$

**c**  $y = 2x$

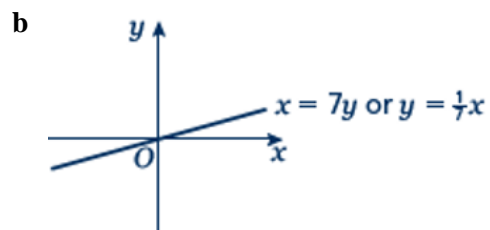
## Answers 8

- 1**   **a**   10.3 cm                      **b**   7.07 cm  
         **c**   58.6 mm                      **d**   8.94 cm
- 2**   **a**    $4\sqrt{3}$  cm                      **b**    $2\sqrt{21}$  cm  
         **c**    $8\sqrt{17}$  mm                      **d**    $18\sqrt{5}$  mm
- 3**   **a**    $18\sqrt{13}$  mm                      **b**    $2\sqrt{145}$  mm  
         **c**    $42\sqrt{2}$  mm                      **d**    $6\sqrt{89}$  mm
- 4**   95.3 mm
- 5**   64.0 km
- 6**    $3\sqrt{5}$  units
- 7**    $4\sqrt{3}$  cm

## Answers 9

1 £77

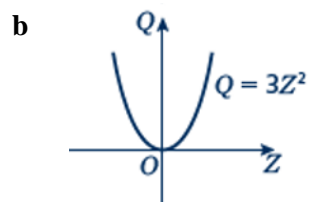
2 a  $x = 7y$



c 91

d 9

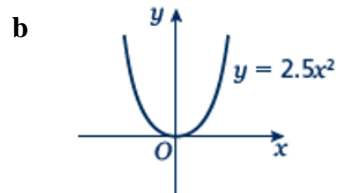
3 a  $Q = 3Z^2$



c 75

d  $\pm 10$

4 a  $y = 2.5x^2$



c  $\pm 6$

5 a 16

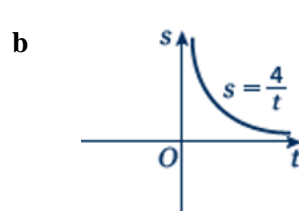
b 100

6 300

7 11.1

8 5

9 a  $s = \frac{4}{t}$

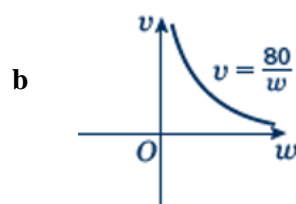


c 4

10 a 2

b 10

11 a  $v = \frac{80}{w}$



c 40

**12** 6

**13 a** 24

**b** 4

**14** 1

**15** 1

**16 a** 0.1

**b** 0.1